Lyapunov Exponents

Lyapunov Exponents are the exponents used to classify how chaotic a system is. These exponents specifically characterize the rate of separation of infinitesimally close trajectories in a dynamical system. It was desired to find these exponents in order to confirm the behavior of our bifurcation diagram. The equation for the change in a trajectory over time is given by,

Eqn 1

where λ is the Lyapunov Exponent. This exponent gives an idea of the predictability of a system by giving the rate of separation of the trajectories. The maximum Lyapunov exponent for a system characterizes its overall behavior. If it is between 0 and 1, not including 0, it is considered chaotic and the greater it is, the more chaotic the system. The max Lyapunov exponent was found for our system using the equation

Eqn 2

to find the exponents for each time and finding the maximum of all these to find the max exponent for a specific mass. This exponent for each mass is plotted below using the function *lyapunovExponents*. It is seen that the exponent is always positive, confirming the chaotic behavior of the machine. This plot has the same scale as our bifurcation diagram, so it can easily be compared. When done so, it is seen that as more chaos is shown on the bifurcation diagram, the exponent increases, suggesting that our bifurcation accurately portrays the system’s chaos.